

Control Lec 8

ر. شهاب

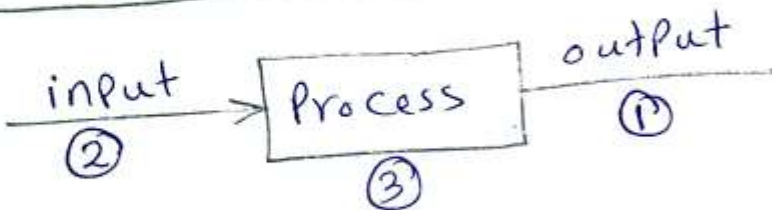
"Control System Design"

Engineering Problems

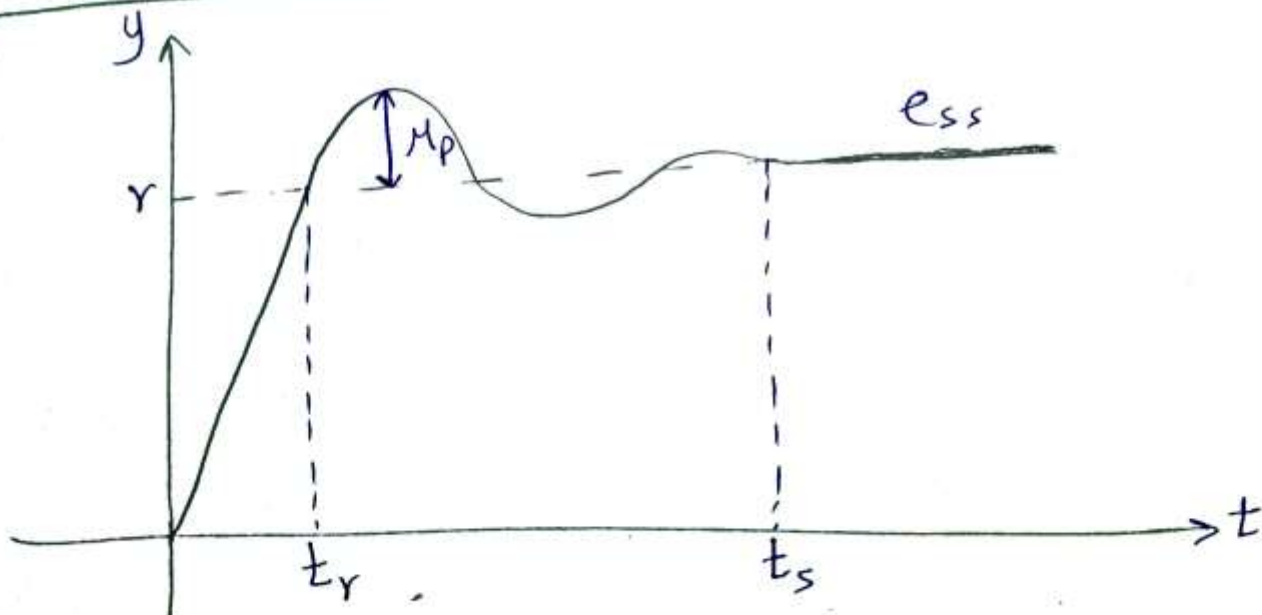
→ Analysis Problem:-



→ Design Problem:-



Control systems Design Specs:-



$$t_r = \frac{\pi - \cos^{-1} \zeta}{\omega_n \sqrt{1 - \zeta^2}} \quad ; \quad t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$M_p = e^{-\frac{\zeta \pi}{\sqrt{1 - \zeta^2}}} \quad ; \quad t_s = \frac{4}{\zeta \omega_n}$$

e_{ss} : steady state error

type 0 : $e_{ss} = \frac{1}{1 + K_p}$

type 1 : $e_{ss} = \frac{1}{K_v}$; type 2 : $e_{ss} = \frac{1}{K_a}$

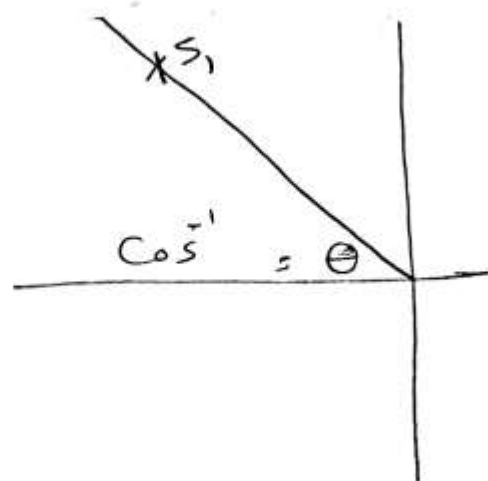
$K_p = \lim_{s \rightarrow 0} G H(s)$; $K_v = \lim_{s \rightarrow 0} s G H(s)$

$K_a = \lim_{s \rightarrow 0} s^2 G H(s)$

* Design Procedure

① desired C.L. clc eqn

$$s_{1,2} = \underbrace{-\zeta \omega_n}_a \pm \underbrace{\omega_n \sqrt{1 - \zeta^2}}_{\omega_d}$$



- Control Design objective :-

Add dynamic components to the system to change the overall system dynamics i.e. change the locations of C.L. poles.

⇒ Remember what we've done in practice :-

- * Requirements
- * Modelling of system dynamics (Analysis)
- * Controller (Add Components)



* Transient Response Compensation

→ lead compensator. → PD

* steady state error compensation

→ lag compensator. → PI.

* For overall Compensation

→ lead-lag compensator. → PID.

* Lead Compensator

at

$$s_d : \angle \phi_z - \angle \phi_p \neq \pm 180^\circ$$

desired

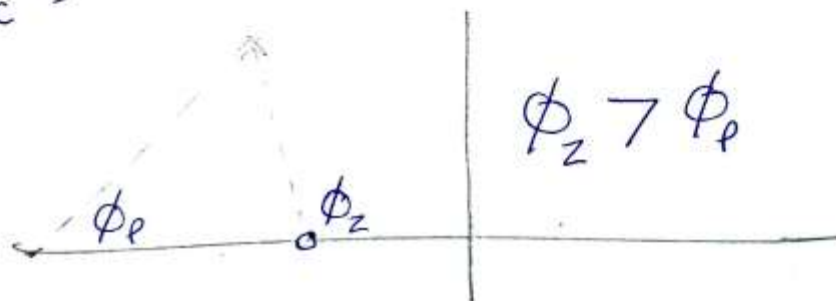
$$\text{at } s_d : \angle \phi_z - \phi_p = \pm 180^\circ$$

By adding zero & pole to the system

$$\angle \phi_{z_c} - \angle \phi_{p_c} = \pm 180^\circ$$

$$\angle GH(s) + \phi_c = -180^\circ$$

Root locus Angle Condition



* Lag Compensator design

$$\textcircled{1} \text{ Calculate } B = \frac{K_c}{K_{un}} * \frac{1.1}{1} \rightarrow \text{safety factor}$$

$\underbrace{K_{un}}_{\text{uncontrolled}}$

K_c : desired value of dc gain.

K_{un} : the system value of dc gain.

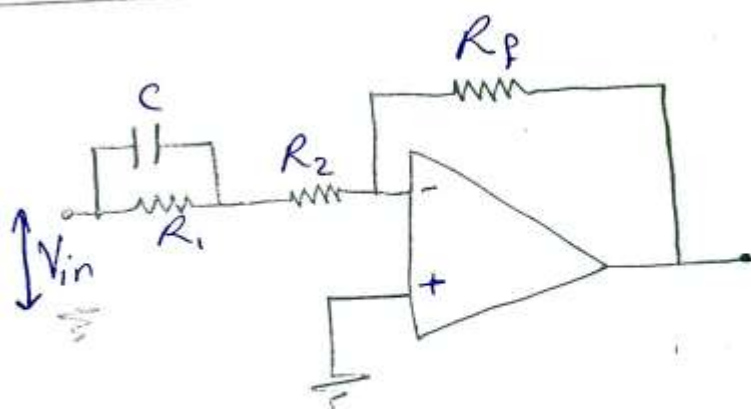
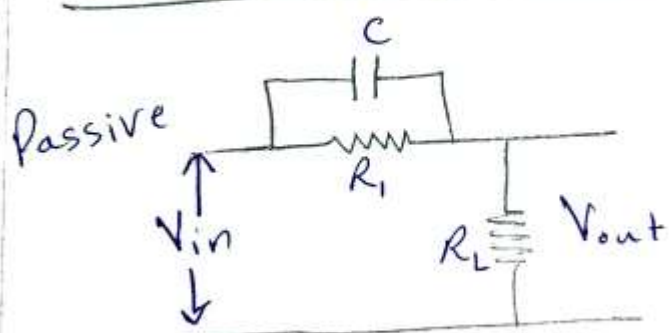
② Assume zero location (Z_c) in the range of 10%. From the 2nd dominant pole of the sys.

$$Z_c = \frac{10}{100} \times p_2 \quad p_2 \Rightarrow \text{2nd dominant pole}$$

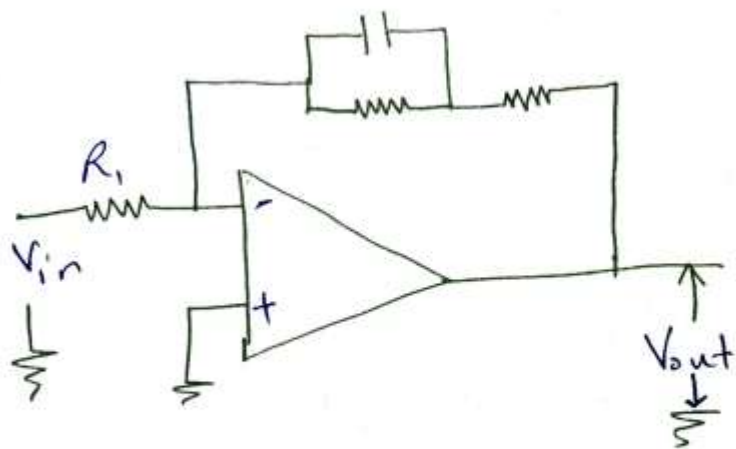
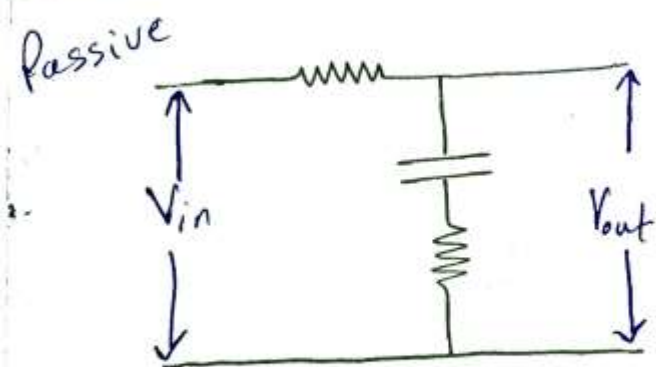
$$P_c = \frac{Z_c}{\beta} \quad K_{dc} = \frac{Z_c}{Z_c/\beta} = \beta$$

of controller

* Lead Compensator



* Lag Compensator



Example

A type 1 system has o.l.t.f ~~o.l.t.f~~

$$GH(s) = \frac{K}{s(s+1)(s+4)} ; \text{ It is desired to compensate}$$

such system to match the following design

~~the~~ specs :-

$$\zeta = \frac{1}{2} \quad e_{ss} \leq 0.1$$

$$\omega_n = 2 \text{ rad/sec}$$

Solution

① desired locations for closed

$$s_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{1-\zeta^2}$$
$$= -1 \pm j\sqrt{3}$$

② From the angle condition

$$\angle GH(s) + \phi_c = -180^\circ$$

$$\angle \frac{K}{s(s+1)(s+4)} = 0 - \angle \frac{-1+j\sqrt{3}}{(-1+j\sqrt{3}+1)} - \angle \frac{j\sqrt{3}}{j\sqrt{3}} - \angle \frac{3+j\sqrt{3}}{3+j\sqrt{3}+4}$$

\downarrow
 $GH(s)$

\downarrow
 $\frac{s}{s+4}$

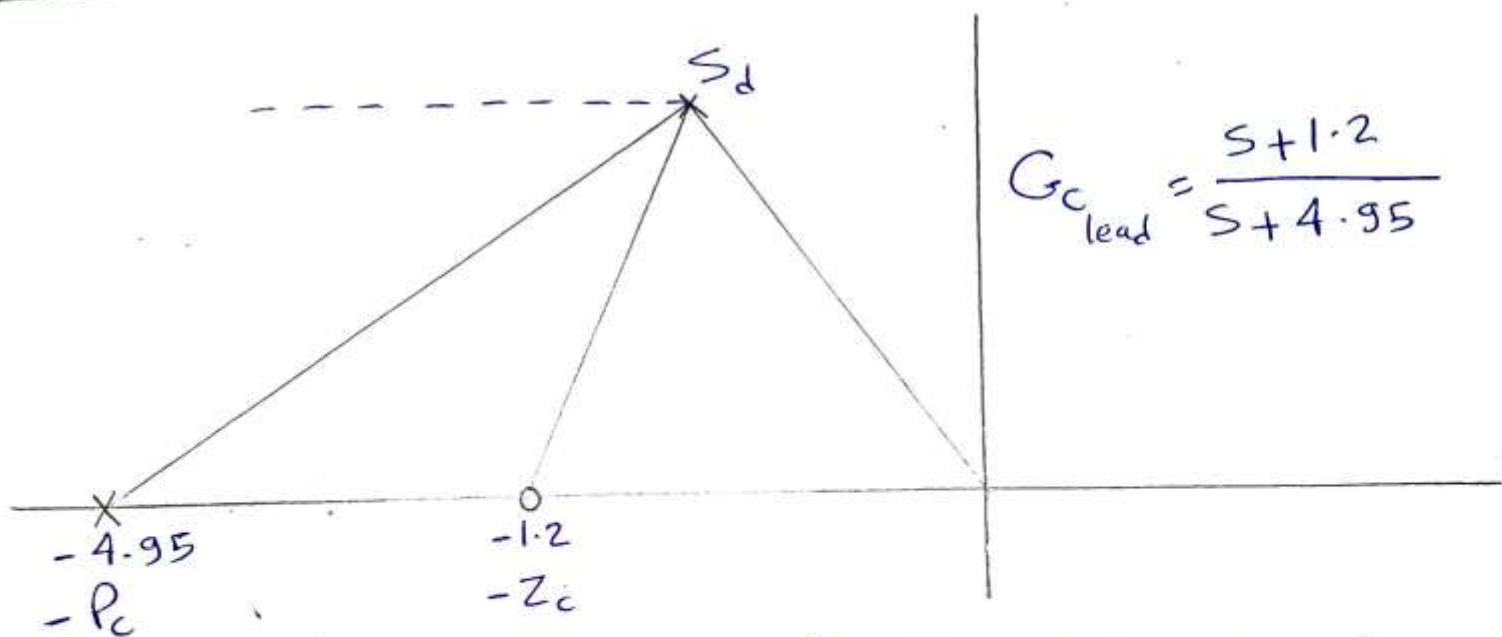
$\boxed{6}$

~~Answer~~

$$-120 - 90 - 30 + \phi_c = -180 \Rightarrow \phi_c = 60^\circ$$

$$\phi_c = 60^\circ$$

③ Determine the locations of Z_c & P_c



• 7. P_c و Z_c هما نقطتان في المستوى العقدي رقم 7

The total T.F

$$G H(s) = \frac{K(s + 1.2)}{s(s + 1)(s + 4)(s + 4.95)}$$

④ Determine K from magnitude condition

$$\|K G H(s)\| = 1$$

$$K = \frac{1}{\|GH(s)\|} = \frac{\|s\| \|s+1\| \|s+4\| \|s+4.95\|}{\|s+1.2\|}$$

$s = -1 + j\sqrt{3}$

$$K = 30$$

← داخل المقياس نضع $(s = -1 + j\sqrt{3})$
ونحسب بالآلة الحاسبة بقا.

⑤ check steady state error

type 1 system $\Rightarrow e_{ss} = \frac{1}{K_v}$

$$K_v = \lim_{s \rightarrow 0} s GH(s)$$

$$K_v = \lim_{s \rightarrow 0} \frac{s(s+1.2) \times 30}{s(s+1)(s+4)(s+4.95)} = 1.81$$

$$e_{ss} = \frac{1}{K_v} = 0.55 \quad , \quad \underbrace{e_{ss} \leq 0.1}_{\text{required}}$$

\Rightarrow to reduce e_{ss} Add lag Compensator
desired $e_{ss} = 0.1 \Rightarrow K_v = \frac{1}{e_{ss}} = 10$

*lag Compensator design

① $K_{un} = 1.82$, $\underbrace{K_c = 10}_{\text{Controlled}}$

$$\beta = \frac{10}{1.82} \times 1.1 = 6.04$$

$\underbrace{\hspace{1cm}}_{\text{safety factor}} \quad \boxed{8}$

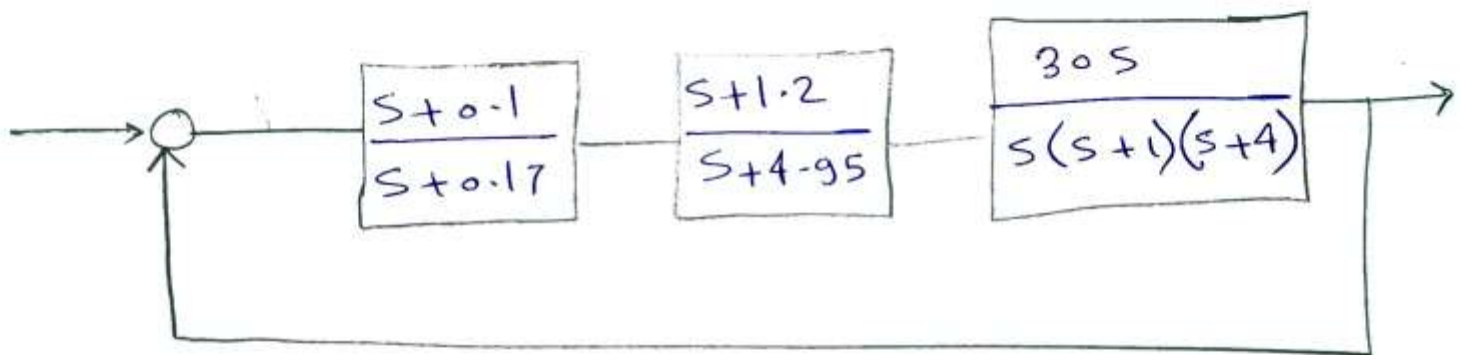
② locations of Z_c and P_c

$$Z_c = \frac{10}{100} \times P_2 = 0.1 \times -1 = -0.1$$

$$P_c = \frac{Z_c}{\beta} = \frac{-0.1}{6.04} = -0.017$$

$$G_{c \text{ lag}} = \frac{s+0.1}{s+0.017}$$

$$GH(s) = \frac{30(s+0.1)(s+1.2)}{s(s+0.17)(s+1)(s+4)(s+4.95)}$$



check e_{ss}

$$\lim_{s \rightarrow 0} s GH(s) = \frac{30s(s+0.1)(s+1.2)}{s(s+0.17)(s+1)(s+4)(s+4.95)}$$

$$e_{ss} = 0.09$$